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AN INTRODUCTION TO "GRAVITATION AND RELATIVITY"*

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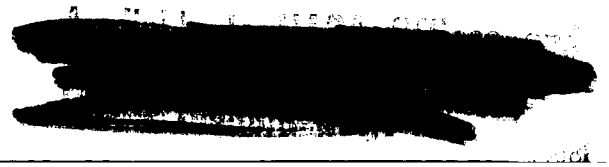
* This table of contents is provided as a reference due to the several citations of various chapters in the Introduction. Many of these chapters have been circulated previously in a preliminary preprint form.



19. A. Nordsieck, Progress in Astronautics in Rocketry, ed. R.E. Robertson and J.S. Farrior, 8, 435 (1962).

20. A.A. Michelson and E.W. Morley, Am.J.Sci. 34, 333 (1887).

21. J.P. Cedarholm, G.F. Bland, B.L. Havens, and C.H. Townes, Phys.Rev.Let. 1, 342 (1958).

22. K.C. Turner and H.A. Hill, Bull.Am.Phys.Soc. 8, 28 (1963).

23. W.F. Hoffmann, Bull.Am.Phys.Soc. 8, 29 (1963).

24. J.E. Faller, Bull.Am.Phys.Soc. 2, 29 (1963).

25. A.R. Sandage, Astro.J. 133, 355 (1961).

26. M.L. Humason. N.U. Nayall, and A.R. Sandage, Astro.J. 61, 97 (1956).

27. H.-Y. Chiu, "Supernovae, Neutrinos, and Neutron Stars," Preprinted: NASA Institute for Space Studies, New York, 1963. (To be published in Annals of Phys.)

28. W.E. Thirring, Annals of Phys. 3, 91 (1958).

29. R.P. Feynman,"Quantization of General Relativity," Lecture series (Recent Developments of Contemporary Physics) given at Yeshiva University, Sept. 24, 1962. (Unpublished).

30. H. Salecker and E.P. Wigner, Phys.Rev. 109, 571 (1958).

Gravitational phenomena and theory are most commonly discussed in the context of general relativity, the preemminent theory of gravity. General relativity is to be distinguished from the special theory of relativity which describes the kinematics of motion. We consider special relativity to be well established by a variety of macroscopic and microscopic experiments. It is a building block presupposed as an implicit ingredient of all theories of physics.

General relativity includes a philosophical extension of many of the concepts of special relativity and, for special examples not involving gravitational fields or accelerated reference frames, reduces to the predictions of special relativity. Beyond this, the two theories are very dissimilar in complexity, scope and meaning. The contents of general relativity are neither completely understood nor completely established. Alternative theories exist with distinct predictions and claims to validity.

This volume emphasizes overriding principles which determine the content and form of the theories of gravity and those experiments and observations which guide and verify the form and are predictions of these theories.

This introduction provides a general discussion of the subject and a brief survey of the contents of the subsequent chapters.

1. Theoretical Foundations of General Relativity as a Theory of Gravitation

1.1. The Principle of Equivalence

After an extensive study of the invariance properties of Maxwell's equations under Lorentz transformations, and of the nature of the null result of the Michelson-Morley experiment, Einstein postulated two basic principles upon which the geometrical structure of space and time are established.[1] These two principles are now known as the principle of equivalence and the principle of covariance. The two principles as Einstein stated them are as follows:

The Principle of Equivalence: "The laws of physics must be of such a nature that they apply to systems of reference in any kind of motion."

The Principle of Covariance: "The general laws of nature are to be expressed by equations which hold good for all systems of coordinates, that is, are co-variant with respect to any substitutions [of coordinates] whatever (generally covariant)."

Before we proceed to discuss further how the field equations are derived, we must examine these two principles in greater detail.

In Chapter 1, Dicke subdivides the principle of equivalence into two principles, one called the strong principle, and the other called the weak principle. The weak principle has its

origin in the following observation. If the gradient of the gravitational field times the dimension of a physical laboratory, or the space-time distance over which experiments can be performed, is much less than unity, then any gravitational effects on the laboratory can be transformed away by letting this physical laboratory fall freely. This characteristic of gravity manifests itself in Riemannian geometry through the possibility of always obtaining a coordinate system in which the components of the affinity are transformed to zero locally. The notion of affinity is discussed in Chapter 2, p.26.

The weak principle of equivalence states this local equivalence of gravitational and acceleration effects:

All material particles in motion follow the same trajectories in space-time, provided their initial conditions (initial position and velocity) are the same.

On the other hand, the strong principle of equivalence is not restricted to the comparison of observations locally. This principle can be stated as follows: If one does experiments to verify any physical laws in a free falling laboratory, one observes the same sets of physical laws independent of the space-time locality one chooses, and independent of the strength of the gravitational field, provided the strength of the gradient of the gravitational field satisfies the restriction stated above.

These two forms of the principle of equivalence seem to differ

from Einstein's original statement. When Einstein proposed his formulation of this principle, the structure of matter was not understood as it is today. In his statement that "the laws of physics must be of such a nature that they apply to systems of reference in any kind of motion," he tacitly assumed that one knows how to transform the laws of physics (discovered or yet to be discovered) to any system in arbitrary motion. However, only if one first assumes a geometrical structure of the space-time continuum (Chapters 1,2,3,4) is it possible to write down the laws of physics in precise language which is meaningful in systems undergoing arbitrary motion. Only then is it meaningful to use Einstein's statement of the principle of equivalence. On the other hand, the two statements presented by Dicke have direct, operational meaning.

The weak principle of equivalence is directly verified to an extrememly high precision by the Eötvös experiment as performed originally by Eötvös[2] and more recently by Dicke.[3] Their experiments verify the following: The ratio of the inertial to the gravitational mass of nearby objects composed of various materials is essentially the same to an accuracy of a few parts in 10^9 (Eötvös) and five parts in 10^{12} (Dicke).

Two conclusions may be drawn on the basis of Eötvös's experiment:

1) All material bodies describe unique trajectories in space

and time when affected by gravitational fields only, provided the restrictions on the strength of the gradient of the gravitational field described earlier remain true.* This is the weak principle of equivalence. It provides a basis for establishing the structure of space-time.

2) The strong principle of equivalence can be inferred to a limited accuracy. That is, one can infer within some limits that the laws of physics (e.g. the ratios of the strengths of fundamental forces) are the same throughout space-time. This is done by the following argument.

If the ratios of various interaction strengths should vary from place to place, then so would the ratios of their contributions to the inertial mass of different objects vary with position. Such position dependent mass-energy would lead to an anomalous force which would be different for materials exhibiting different ratios of mass-energy content due to these various interactions. The experimental absence of such anomalous forces implies the constancy of the interaction ratios to a precision depending on their fractional contribution to the total mass of the various materials. In particular, the contribution of strong interactions, electromagnetic interactions, weak interactions and gravitation to the energy of a body is roughly $1:10^{-2}:10^{-12}:10^{-40}$ for an atom (the contribution from the strong interaction is normalized to unity). For a macroscopic body of mass around one gram and density

*This restriction is necessary since the trajectory of a spinning test particle is different from the trajectory of ordinary particles in the presence of a non-uniform gravitational field.[4]

unity the total gravitational energy is around 10^{-8} ergs and the above ratios become $1:10^{-2}:10^{-12}:10^{-29}$.

The Eötvös experiment demonstrates that the inertial and gravitational masses of material bodies have the same ratio to an accuracy of 5 parts in 10^{12}. Hence to great accuracies, the constancy of the electromagnetic and the strong interactions must follow the strong principle of equivalence. Nothing can be said about the weak interaction and the gravitational interaction. Hence, the Eötvös experiment rules out the possibility that the coupling constants of strong interactions and electromagnetic interactions vary with time and position. It does not rule out the variability of the weak coupling constants and the gravitational coupling constant. This possibility may be closely connected with Mach's principle (Chapters 8 and 15).

One way to test the validity of the equivalence principle in gravitation is to observe the trajectories of objects whose self-gravitational energy is large. Only for astronomical bodies is this possible. For example, for the sun the above interaction strength ratios are $1:10^{-2}:10^{-12}:10^{-6}$. For Jupiter the ratios are $1:10^{-2}:10^{-12}:10^{-8}$. A study of the trajectory of Jupiter might lead to more precise limits in the applicability of the strong principle of equivalence to the gravitational self-energy.

1.2. The Principle of Covariance

This principle states: "The general laws of nature are to

be expressed by equations which hold good for all systems of coordinates, that is, are co-variant with respect to any substitutions whatever (generally covariant)."[1]

There are two interpretations of this principle:

1) We may interpret this principle as the statement that a coordinate system is just a particular choice of imaginary lines in space and time whose intersections characterize events. Hence, the choice of coordinate systems has nothing to do with the contents of the theory. This is a restricted sense of covariance, and in a sense is an empty statement; for any theory may be made to be covariant without increasing its content. For example, Maxwell's equations may be written in a form which appears to be generally covariant (Chapter 11, Eq.34), without enlarging its content as long as the metric is restricted to that of a flat space. Then the metric can always be restored to the familiar diagonal Lorentz form by an ordinary coordinate transformation. In this restricted sense, all laws of physics may be written in a covariant form without introducing any new physics (Chapter 9) and the principle of covariance is an empty statement.

2) What Einstein had in mind when he stated the principle of covariance is entirely different. What he meant is: The laws of physics must be of such a nature that they apply to all coordinate systems compatible with Riemannian geometry not merely those with a flat space metric. Thus, in the case of the covari-

ant form of electrodynamics (Chapter 11, Eq.14) the subsidiary condition that the space be intrinsically flat (that is, that one can find a coordinate transformation such that the line element has the form: $ds^2 = \pm \delta_{ij} dx^i dx^j$) must be removed and a more general geometry be permitted, which must be determined by the field equations. This goal, however, has not been achieved. Hence it is not clear what we mean by, for example, "electrodynamics in a curved space."

In addition, it is not clear how useful it is to require that the fundamental, microscopic laws of physics be applicable to spaces of arbitrary curvature. The microscopic laws of physics usually apply to a spatial dimension of around 10^{-8} cm (atoms) or to 10^{-13} cm (nucleus and fundamental particles). Unless the spatial variations of curvature are so large that the curvature changes substantially over such small distances one can always transform to a coordinate system in which the metric is locally flat and free from gravitatitional effects. Hence it may not be necessary to require the more general principle of covariance in dealing with microscopic phenomena.

Moreover, the requirement of covariance introduces additional complications. The solutions of the Einstein field equations must admit a general coordinate transformation in order to be generally covariant. This is reflected by the fact that the ten

equations satisfy four identities (Chapter 4). These are the conservation laws of energy and momentum expressed in the form

$$p^{\mu\nu}{}_{;\nu} = 0 \tag{1.1}$$

Hence there is an arbitrariness in the solutions. This arbitrariness prevents us from obtaining a definite solution from the field equations. In order that a definite solution be obtained, four additional equations must be set up for the $g_{\mu\nu}$. These four equations must be in a non-covariant form and they must be chosen so that, if we start with any set of $g_{\mu\nu}$, we can satisfy them by merely carrying out a coordinate transformation. Such equations are known as <u>coordinate conditions</u>.

Such non-covariant coordinate conditions cause a great deal of concern among relativists. Does this mean there exist a set of preferred coordinate systems in general relativity, which are not contained in Einstein's theory but must be obtained from other arguments? On this question the opinion among relativists is much divided. Fock[5] suggested that a set of harmonic coordinate conditions expressed in the form

$$(\sqrt{-g}\, g^{\mu\nu})_{,\mu} = 0 \tag{1.2}$$

together with certain conditions at infinity (such as no incoming gravitational radiation) determine a preferred set of coordinates. His argument is as follows: In flat space there exists a preferred

set of coordinate systems, namely, the Cartesian coordinate systems. For an inertia system, the Cartesian coordinate system is determined uniquely up to a Lorentz transformation. Similarly, in the case of an isolated system of masses there exists a coordinate system, namely the harmonic system (obtained with the coordinate condition Eq.(1.2)), which is determined uniquely up to a Lorentz transformation if suitable supplementary conditions are imposed.

Many other relativists (for example, Anderson, Chapter 9) argue that, once the curvature of space is determined by Einstein's equations, the choice of coordinate systems is entirely arbitrary; it is just a way to draw imaginary lines in space-time to characterize events. It has nothing to do with the structure of the physical theory. Hence there are no preferred coordinate systems. This controversy has not been settled and remains one of the outstanding questions of general relativity.

Once the curvature properties of space-time are determined from Einstein equations, only the restricted sense of the covariance principle may apply to coordinate systems, and, as mentioned above, this may be the only part of the principle of covariance which is applicable to microscopic theories of matter.

1.3. Identification of Gravitational Fields with the Geometrical Structure of Space-time

To the accuracy of the Eötvös experiment, all material bodies follow unique trajectories in space-time, if gravitational forces alone act. Would not this imply that the gravitational field is a

property of the space, independent of the detailed structure. That is, can one replace gravitational fields by geometrical structure of space-time. This can and has been done for gravitation theory. Furthermore, to date, gravitation theory remains the only one which has been geometrized.

One way to achieve the geometrization of gravitation theory is to define the geometry in such a way that particle trajectories coincide with unique curves contained in the geometry. One choice is to choose the geometry such that the extremal paths coincide with particle trajectories. Then the gravitational field is replaced by the curvature of space and the equations of motion are equations for geodesics. This is the approach taken by Einstein. However, there are many ways of defining unique curves in geometry without their being geodesics. This is discussed in detail in Chapter 1. In either case the invariance properties of physical quantities supply us with a method for relating quantities measured in one coordinate system to those measured in another. The invariance and transformation properties of geometrical quantities are discussed in Chapter 2.

1.4. Classical Measurements of Space-time Distances

In the geometrized theory of gravitation, the gravitational field is replaced by the curvature of space, which is characterized by a set of metric coefficients $g_{\mu\nu}$ (metric tensor). They are

defined in such a way that the line element ds (the distance between two neighboring space-time points) is given by

$$(ds)^2 = g_{ij} dx^i dx^j \tag{1.3}$$

where summation over repeated indices is understood. In order to use this notion of distance operationally one must have a scheme for measuring distances. This is necessary not only for infinitesimal distances as described in Eq.(1.3) but also for large distances.

Classically, Einstein used material rods and clocks to perform space and time measurements. In a frame in which the rods and the clocks are at rest, the two measuring instruments perform orthogonal measurements, that is, a clock measures the distance along the time axis, and a rod measures the distance along the spatial axis. If the strong equivalence principle is to hold exactly and precisely then all coupling constants will indeed be constant and will yield unambiguous results. Then rods and clocks could be relied on to be unchanged from place to place and the comparison of measurements (such as the velocity of light) from place to place would have a clear meaning.

However, according to one version of gravitational theory (Dicke, Chapter 8), some of the fundamental constants are not really constant, but may vary with time. The ratio of distance

measurements performed by using a material rod (composed of atoms) to those performed by counting the number of wavelengths of a red spectral line of Kr^{86} is proportional to $e^2/\hbar c$ (Chapter 3). Hence, if the strong principle of equivalence is in doubt (Chapter 1 and 8) all measurements performed by using clocks and different kinds of rods will yield ambiguous results. Statements like "the velocity of light has a constant numerical value measured locally in all frames of reference" will not be meaningful.

On the other hand, it is not necessary to use clocks and rods at all to measure space-time intervals since in the space-time continuum a purely space-like or time-like interval will not be purely space-like or time-like in a moving system. It is possible to combine the measurement of space and time intervals into a single operation. This operation can be defined entirely in terms of light beams and particle trajectories independent of the constitution of matter. The constancy of the velocity of light measured in any local system then becomes a postulated principle by which measurements are interpreted. This was first suggested by Weyl[6].

In Chapter 3, Marzke and Wheeler demonstrate one way by which space-time distance measurements may be carried out over a long distance in the presence of gravitational fields. A world tube is formed, in which light scatters back and forth along the tube. They have shown that the number of scatterings will yield

an invariant measurement of a space-time interval.

1.5. The Einstein Field Equations

The Einstein field equations restrict the types of geometries that are physically realizable. They also provide a means for constructing the geometry when the stress tensor (the source of gravitational field) and suitable boundary conditions are given. From the field equations we can also derive equations of motion of test particles which must reduce to the Newtonian equations of motion in the case of weak fields and low velocities.

That the stress-energy tensor $T_{\mu\nu}$ is the source term for the gravitational field is implied by the results of the Eötvös experiment. This experiment indicates that the source of gravitational interaction is the total mass energy. Any matter distribution will have in addition to the rest energy of matter, also the stress energy. (Rigid bodies have no invariant meaning since a strain wave cannot be transmitted instantaneously.) Hence the matter distribution must be replaced by the stress-energy tensor $T_{\mu\nu}$, which includes the matter distribution as one of its components.

With the above mentioned requirements that the theory be generally covariant, that is involve only the metric tensor and various geometrical objects obtained from its components and derivations of these, that the stress-energy tensor be the source of gravitational effects, and that the equations be the lowest

order possible, the only field equations which follow are the Einstein field equations (Chapter 4).

2. Experimental Foundations of General Relativity Theory and Other Theories

General Relativity theory, and other theories of gravity, are based on one or more of the following principles:

1) The equivalence principle

2) The isotropy of space-time

3) The principle of covariance

4) The constancy of electric charge independent of its velocity and the consequent Lorentz invariance of special relativity

5) The justification of using Riemannian geometry (in fact, any geometry at all) to describe the space-time structure of the world

Although the underlying isotropy of space-time is implied by the equivalence principle, it is listed as a separate principle here to emphasize the possibility of a local anistropy due to a local anistropy of mass distribution. This is discussed in more detail below and in Chapter 13.

Before we discuss these five principles, we remark that their validity must be interpreted in an operational sense. That is, their validity or invalidity must be demonstrated experimentally. If the violation of a principle occurs in such a way that in principle no experiments (even gedanken experiments) may tell us about the violation, we do not consider that this violation takes

place.

With this in mind, we discuss the experimental support for these principles.

2.1. & 2.2. The Equivalence Principle and the Principle of the Homogeneity and Isotropy of the Space-time Structure

We have already discussed the theoretical content of the equivalence principle in detail. This principle implies the principle of homogeneity and isotropy of space-time. However, to the extent that the strong principle of equivalence may be violated (Chapter 1 and 8 and the Introduction, p. 4), the homogeneity and isotropy of space may require special attention.

Mach's principle requires that the inertial properties of matter be determined by the distribution of matter in the universe. A naive interpretation of Mach's principle is that the inertial property of matter depends on local distribution of matter, and therefore, it is expected to depend on the direction of motion. However, such anistropy cannot be observed (Chapter 7).

Beltram-Lopez, Robinson, and Hughes[7] performed experiments to determine if the inertia of matter depends on the spatial direction (Chapter 6). The local source of mass anistropy is the sun and the galaxy. These experiments have been done both with paramagnetic resonance absorption measurements of the Zeeman splitting in chlorine and oxygen and by nuclear magnetic resonance in lithium. In the case of Zeeman splitting, an atomic electron with non-zero orbital angular momentum moves in different directions with respect

to an external magnetic field in different magnetic substances. Hence, if the mass depends on the direction of motion, the energy difference between different magnetic substances will depend on the mass anistropy. A similar situation occurs with the different nuclear magnetic states in the lithium experiment. Because of the much higher binding energies in the nucleus compared with those of the atomic electrons, the lithium experiment is the more precise one. The results of this experiment place an upper limit of 1 part in 10^{22} to the anistropy of mass.

2.3. The Principle of Covariance

The theoretical meaning of the principle of covariance has been discussed above on page 5 . The first statement of this principle reflects the arbitrariness of the mathematical representation of space in terms of coordinates. It is a consequence of the possibility of making and interpreting measurements at all, for if the particular method of labeling the phenomena should affect it, no meaningful measurements could be made.

The second statement of the principle of covariance discussed above has physical content to the extent that it restricts the possible structure of theories to those which can be written in a generally covariant form equally valid in Riemannian geometry with an arbitrary curvature. Gravitational theory has been written in this form. Its success in interpreting gravitational phenomena may be regarded as the experimental basis for the validity of the second statement of the principle of covariance. Unfortunately,

no other theories of physics have been so formulated and as mentioned on page 6 , it may not be experimentally meaningful to so formulate them.

2.4. The Constancy of Electric Charge Independent of its Velocity

The constancy of electric charge independent of its velocity is implied by the invariance of Maxwell's equations under a Lorentz transformation. This constancy is the basis of electrodynamics as well as of special relativity, from which general relativity evolved. It has been established to a high degree of precision as a by-product of several experiments carried out to measure the electron-proton charge ratio. Some of these experiments are described by Hughes in Chapter 13. In one of these, performed by Zorn, Chamberlain, and Hughes[8], the deflection of a neutral molecular beam passing through an electric field perpendicular to the direction of motion was measured and found essentially to be zero (aside from induced polarization effects). From the sensitivity of the apparatus for a beam of Cesium atoms the equality of electric and proton charge is established to one part in 10^{19}. In other experiments, Hillas and Cranshaw[9] established the equality to one part in 10^{21} with Argon in a gas efflux experiment. These experiments not only demonstrate that the electric charge for electrons and protons are equal and and opposite but also as a byproduct, demonstrate that the electric charge is a constant independent of its velocity to an extremely high degree of precision.

The importance of this experiment to the question of velocity dependence of electric charge comes about in the following way.

In a hydrogen atom the proton and the electron move around their common center of mass. Because of its lighter mass, the velocity of the electron is higher than that of the nucleus by many orders of magnitude. In a many electron system the motion of the nucleus is even smaller. In the case of argon ($z = 18$), the velocities of the k-shell electrons are around 10^9 cm/sec and the charge equality limit is $1/10^{21}$. If we use $(\frac{v}{c})^2$ as a parameter for the dependence of electric charge with respect to its velocity, we find the coefficient of the dependence is less than one part in 10^{17}. This may be regarded as the accuracy to within which the electric charge is independent of the velocity.

2.5. The Justification of Using Riemannian Geometry to Describe the Space-time Structure of the World

One of the most important postulates on the structure of space-time is that measurements should have an invariant meaning. If two identical masses, or two identical meter sticks, are taken along two different arbitrary, closed paths starting at the same point, will their mass or their length depend on the path taken?

The length of a material rod depends on the value of the Bohr radius r_b:

$$r_b = \frac{\hbar^2}{me^2} \tag{2.1}$$

To the extent that the strong equivalence principle applies to electromagnetic theory, e and $\hbar$ are constants of nature. What about m?

In Chapter 4, Wheeler argues as follows: In the case of a many electron atom, if all electrons are identical, Pauli's exclusion principle applies and the atom will not collapse. If, for some reason, the mass of one electron is different from the rest, then Pauli's exclusion principle will not apply to it and it will cascade down to its lowest quantum state. The atom will then collapse.

The precision to which we can conclude that the masses of the electrons are identical is limited by the quantum uncertainty in the energy of the individual electrons due to the finite life of the atom. Let the energy of a certain inner shell electronic state be E, with an uncertainty ΔE, which may be computed in the following way:

$$\Delta E = \frac{\hbar}{t} \tag{2.2}$$

where t is the time since the atom was formed (during which these inner shell electrons have undergone no transition). Let $t = 10^6$ years (a short time compared to the age of the universe). Then

$$\Delta E = 10^{-40} \text{ ergs}$$

$$\frac{\Delta E}{mc^2} = 10^{-34} \tag{2.3}$$

This is roughly the extent that the mass of an electron is constant independent of the path it has taken in the past and the extent to which, on these grounds at least, the use of Riemannian geometry is

justified.

3. Experimental and Observational Tests of Gravitation Theory

For a theory of such seemingly pervasive importance underlying the structure of space-time and, as is sometimes suggested, perhaps of elementary particles as well, general relativity has led to remarkably little successful experimental research. The few predictions the theory does make about observable phenomena require an almost impossible precision for any decisive measurement. Such precision has been realized for only three experiments in the past: Analysis of the orbit of the planet Mercury for a small relativistic precession of the perihelion of the orbit, gravitational bending of starlight passing by the sun, and the red shift of spectral lines emitted and observed at two different gravitational potentials. It is currently being being sought for two more: Precession of a gyroscope in the field of the rotating earth and gravitational radiation. It is therefore no surprise that gravitation has met with limited interest in the past as an object of experimental research.

In the last few years, there has been an increase in activity and interest in the field. This has come about variously by the availability of new and precise experimental techniques (e.g. earth satellites, precise electronics systems); by a broadening base of theoretical work in gravitation theory, and by an increasing feeling

among some physicists of the ultimate role gravity may play in other fields of physics, such as the structure of elementary particles. The new experimental work is proceeding along several lines:

1) A refinement of the measurements of the "three famous checks" of general relativity: the precession of the perihelion of Mercury, the bending of starlight by the sun, and the gravitational red shift.

2) Attempts to measure previously undetected general relativistic effects, in particular the Lense-Thirring precession of a gyroscope in the gravitational field of the earth, and the gravitational radiation from oscillating laboratory or astronomical masses.

3) Fundamental precision measurements which are prerequisite to any theory of gravity: the experiments concerning the equivalence of gravitational and inertial mass, mass isotropy and charge equality.

4) Attempts to detect effects not predicted by general relativity but suggested by alternative theories: gravitational "ether drift"; time varying gravitational "constant"; the interaction of scalar matter waves with the solar system.

5) Incorporation of general relativity into the astrophysical models of very dense stars with the hope that they will lead to observable implications.

6) Search for cosmological effects such as the overall curvature and closure of space.

3.1. Refinements of Measurements of the Three Famous Checks

These experiments are not emphasized in the subsequent chapters of this book, so we will go into some detail here.

The field equations of general relativity are non-linear. The principle of superposition cannot be applied to gravitational fields without caution. To discuss general relativistic effects, one examines the response of very small infinistesimal bodies (test bodies) or light rays whose perturbing effect on the gravitational field is negligible. In particular, the three experimental tests to be discussed here involve the behavior of such test bodies for the Schwarzschild solution of Einstein's equations.

The Schwarzschild solution is an exact solution of Einstein's field equations with the following properties. It is:

1) Time independent (static)

2) Spherically symmetric around some point

3) Reducible to that for a Newtonian gravitational field for a mass point and a flat geometry at large distances.

In other words, the Schwarzschild solution is the solution of the Einstein Field equations for the gravitational field produced by a point mass with the above properties [1) - 3)].

The Schwarzschild solution of the line element ds in the

so-called isotropic coordinate system, r', θ, φ, t, is

$$ds^2 = (1 + \frac{m^*}{2r'})^4 (dr'^2 + r'^2 d\theta^2 + r'^2 \sin^2\theta d\varphi^2) - \frac{(1 - m^*/2r')^2}{(1 + m^*/2r')^2} c^2 dt \quad (3.1)$$

where m* is the geometrized mass related to ordinary mass m by the relation

$$m^* = \frac{Gm}{c^2} \quad (3.2)$$

In the weak field limit for which $m^*/2r' << 1$ we can replace r' by an expression involving the spherical coordinate r and express the coefficients for the terms in this line element in the form of a power series expansion in $(\frac{m^*}{r})$:

$$ds^2 = -(1 - \frac{2\alpha m^*}{r} + \frac{2\beta (m^*)^2}{r^2}) c^2 dt^2 + (1 + \frac{2\gamma m^*}{r} \cdots\cdot)(dr^2 + r^2 d\Omega^2) \quad (3.3)$$

The relation of isotropic coordinates r', θ, φ, t to the spherical coordinates r, θ, φ, t (which reduce to the ordinary three dimensional spherical coordinate system in the weak field limit) is:

$$r = r'(1 + \frac{m^*}{2r'})^2$$

$$r' = \frac{1}{2}\{(r^2 - 2m^*r)^{\frac{1}{2}} + r - m^*\}$$

$$\theta = \theta;\ \varphi = \varphi;\ t = t \quad (3.4)$$

In the weak field limit these two coordinate systems have the same meaning. In the strong field limit the spherical coordinate r has

no operational meaning.

For the Schwarzschild solution the coefficients α, β, γ in Eq.(3.4) are unity. They are inserted into Eq.(3.3) in order to facilitate determining what features of the line element various experiments have a bearing on and to permit distinguishing between the predictions of general relativity, for which these coefficients are unity, and any other theory for which the line element may be expanded in a similar form with coefficients not necessarily equal to 1.

The value of α is determined to be unity by the requirement (3) above that Einstein's theory yield an inverse square Newtonian gravitational field at great distances. This term also accounts for the gravitational red shift. The difference in clock rates, hence in atomic spectra frequencies at different distances from a mass point is given by

$$\frac{\Delta\nu}{\nu} = \alpha\frac{Gm}{rc^2} \tag{3.5}$$

The angle of deflection of light at the limit of the sun is given by

$$\theta = 2(\alpha + \gamma)\frac{GM_o}{R_o c^2}$$

$$= 0".82\ (\alpha + \gamma) \tag{3.6}$$

and in Einstein's theory $\alpha = \gamma = 1$. This gives

$$\theta = 1."74$$

The value of α is given by the Newtonian inverse square law of gravitation and may also be inferred by the principle of equivalence and the results of observations of the gravitational red shift. Arguments have also been made to derive the bending of light (which involves γ) using the principle of equivalence in a flat space geometry.[10] However, this derivation requires additional assumptions about the relationship of accelerating coordinate systems separated in space which are not implied by the Eötvös experiment. It has been shown in at least one case[11] that it is possible to formulate a theory which satisfies the principle of equivalence and predicts a value for the bending of light which differs from general relativity. This case, the theory of Dicke and Brans, implies a value for the light bending given by

$$\theta = \left(\frac{3 + 2\omega}{4 + 2\omega}\right)\frac{4Gm}{R_{\odot}c^2} \tag{3.7}$$

where ω is a coupling constant for a scalar field. Its value is determined to be greater than 6 from the observation of the relativistic precession of the perihelion of Mercury.

The prediction of general relativity for the precision of the perihelion of a planet can also be given in terms of the coefficients α, β and γ. In this case the predicted precession is multiplied by the factor $\left[2\alpha(\alpha + \gamma) - \beta\right]$.

Of these three tests of general relativity, only the observation

of the precession of the perihelion of Mercury predates the theory of general relativity. Discrepancies in Mercury's orbit have confounded astronomers since the middle of the 19th century. In 1859 LeVerrier noted an unexplained excess in the advance of the perihelion of Mercury over the theoretical value of approximately 39 seconds of arc per century.[12] Twenty-five years later Newcomb determined this discrepancy to be 43 seconds of arc per century.[13] The most extensive modern analysis is that of Clemence [14] which yields a value of 43.06 $\pm$ 0.06 sec of arc/century. This is to be compared with the general relativistic prediction of 43.03 sec.

This furnishes the best support for general relativity, both because of the impressive agreement between experiment and theory and because the theoretical prediction of the precession, unlike the other two tests, depends not only on the effect of gravitation on time and length measurements to terms quadratic in the mass but also on the general relativistic modification of the equations of motion of a massive particle. Hence it is all the more important to confirm these results and to be sure that the calculated geometrical and dynamical corrections which total approximately 5600 sec of arc per century are not subject to systematic error.

Unfortunately, this result is unique. That is, it depends on the analysis of a single planet's motion for relativistic precession. Because of the large number of corrections required which may be

subject to systematic error, it is unreasonable to weight this measurement as heavily as the random error would suggest without independent corroboration from accurate observation of other planetary precession. Unfortunately, the relativistic precession of other planets is less by a factor of 10 than that of Mercury. Of these the Earth and Venus have been observed with accuracies of only 50 %. The situation for earth satellites is even worse with the non-spherical shape of the earth producing perturbation on the orbit far in excess of the relativistic precession.

One of the possible perturbing effects on Mercury's orbit is that due to the non-sphericity (oblateness) of the sun. This effect on the precession is given by

$$\frac{\text{Precession angle}}{\text{rev}} = \frac{6\pi}{5}\epsilon\left(\frac{R_\odot}{R}\right)^2 \; ; \qquad \epsilon = \frac{\Delta R_\odot}{R_\odot} \tag{3.8}$$

where $\Delta R_\odot$ is the difference between the polar and equitorial radii of the sun. The oblateness of the sun (given by ϵ) has not been measured but is estimated on the basis of a simple model of the rotating sun to produce a negligible precession of Mercury.

Dicke and associates are undertaking a measurement of the oblateness of the sun to determine an accurate measure of this correction. The experiment is discussed briefly in Chapter 1.

The second "classical" test of general relativity in historical order is the observation of the bending of starlight as it passes through the gravitational field of the sun. The theoretical prediction for the bending of starlight both by Newtonian corpuscu-

lar theory and for general relativity is discussed in Chapter 2. Observations have been carried out at a number of solar eclipses since 1917. These experiments, subject to many uncalculable systematic errors, average about 20 % above the theoretical value for the average extrapolated limb shift, that is, the shift in apparent star position for a line of light between star and observer barely grazing the edge of the sun. Unfortunately, these results are not decisive because of an uncertainty of around 25 % in their accuracy. Furthermore, these observations give only the limb shift and do not establish the radialdependence of the bending.

An experiment is being undertaken by one of us (W.F.H.) at Yale to measure the deflection of starlight by the sun during the day without an eclipse. This approach has the advantage of permitting continuous monitoring of star separations while the sun passes through the star field, and of allowing very much increased observing time for isolating systematic errors. In addition, by using photoelectric rather than photographic detection one can eliminate the problems of photographic plate deformation in the determination of the scale function for converting the observed star displacement to angles in the celestial sphere. On the other hand the presence of a bright background sky and bright sun present a difficulty of high background and low signal-to-noise ratio not present in the eclipse observations.

The method of this daytime observation involves the use of a coronograph type of instrument in which direct light from the sun is blocked from reaching any optical elements beyond the first objective lens (Fig. 1). The actual telescope planned eliminates some of the optics shown while retaining the fundamental principle of the instrument. The presence and position of a star is determined by a position-modulated photoelectric device in the focal plane of the telescope which distinguishes the star image from the background of sky light and sunlight scattered in the objective lens. The angular separations of several stars in a small region near the ecliptic are to be observed by pairs during the several days required for the sun to pass through this region. It is expected that an analysis of the shift in the apparent relative position of these stars will provide a considerable improvement in accuracy over the eclipse determinations.

The third classical check on relativity, both in point of time and in importance for distinguishing the prediction of general relativity from other gravitational theories is the gravitational red shift. The theory of this effect is discussed in Chapter 11. The theoretical prediction given by general relativity, and by other arguments involving only the principle of equivalence and Doppler shift is

$$\frac{\Delta\nu}{\nu} = \Delta\left(\frac{Gm}{rc^2}\right) \qquad (3.9)$$

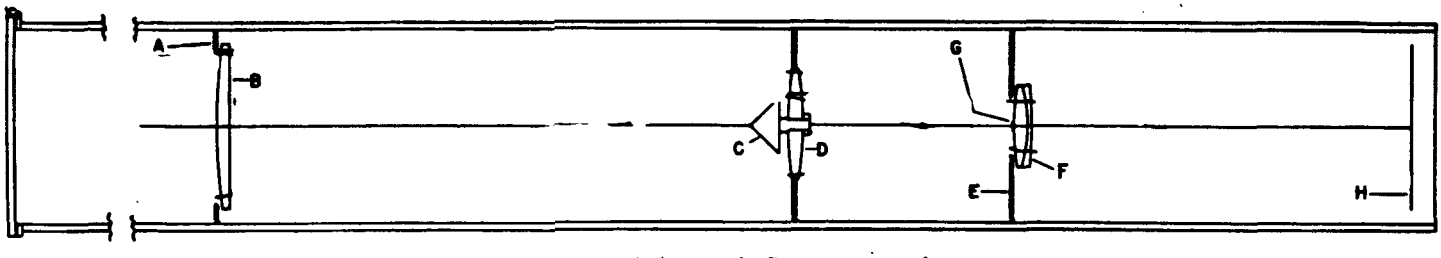


Figure 1. Coronographic Telescope. The oculting cone (C) placed at the focal plane of the primary objective blocks direct sunlight from reaching the subsidiary optics. The lens (D) focuses the rim (A) of the primary objective lens (B) at the field stop (E), which blocks the ring of light scattered and diffracted from the rim (A) of the main objective at (B). The central spot (G) blocks the solar image formed by multiple reflection within the objective lens (B). The focal plane is at (H). (From G.P. Kuiper, editor; "The Sun, Vol. I," The Solar System, Figure 21 p.638, Chicago Univ. Press: Chicago, 1953.)

The gravitational red shift was first measured astronomically for light from the companion of Sirius[15], a white dwarf whose mass is determined from its motion with Sirius and whoses radius is determined from the theory of Chandrasekhar for white dwarfs. The results from these observations are very uncertain both because of the problem of isolating the light of Sirius B from that of Sirius A and because of subsequently recognized spectral line shifts resulting from the details of the pressure and convection in the stellar atmosphere. Further sttempts to measure this effect on stars and the sun have primarily resulted in an acknowledgment of the complexity of the problem of disentangling the gravitational red shift from a variety of Doppler and pressure shifts which occur in the stellar photosphere. Only recently has this difficulty been partly overcome.

The first good red shift measurements were terrestrial. Pound and Rebka[16] using the Mössbauer effect for resonant absorption of gamma rays by Iron-57 for a source at a vertical separation of 74 feet, measured the frequency shift to be 1.05 ± 0.10 times the theoretical value. Subsequently, Brault[17] has measured the red shift in absorption lines in the sun's photosphere for the Sodium D1 line which is free from many of the disturbing effects since the absorption for strong lines is high in the solar atmosphere. He obtained $1.05 \pm .05$ of the theoretical value.

3.2. Measurements of New Relativistic Effects

The second area of current experimental work in gravitation involves the detection of other effects predicted by general relativity but not yet observed. One of these is the effect on the orientation of the spin axis of a gyroscope in the field of the rotating earth.

According to Newtonian theory, such a gyroscope, in the absence of bearing friction and structural asymmeties should point indefinitely in its initial direction relative to "space" as determined by the "fixed" stars. General and special relativity predict a precession of the spin axis which arises from three causes:

1) As the gyroscope is carried around by the rotation of the earth, its orientation should behave according to the parallel transport of a vector in general relativity. The notion of parallel transport of a vector which is discussed in Chapter 2 in connection with the ideas of covarient differentiation and affinity. Such a vector, transported in a circle around a mass center will not return to its initial orientation. This is spoken of as the "geodetic precession."

2) In the vicinity of a rotating mass the inertial frame is "dragged around" slightly by a small fraction of the rotation rate of the mass. Thus a gyroscope, even at rest near the earth

with its axis not coaxial with the earth will experience a changing orientation relative to the fixed stars. This precession is referred to as the Lense-Thirring precession.

3) A spinning object on the earth in motion (due to the earth's rotation) experiencing non-gravitational forces (the bearing supports) will precess according to special relativity.

For a gyroscope on earth at the equation with spin axis normal to the earth's axis, all three effects are approximately the same value, 0.4 seconds of arc per year. For a gyroscope in a satellite, the first two effects are somewhat smaller, depending on the orbit parameters, and the third is absent.

These effects on gyroscope precession are extremely small and hence normally completely obscured by instrumental effects. However, it would be interesting to observe them. Two approaches are currently being undertaken to develop adequately stable gyroscopes for this experiment. One involving a superconductor magnetically supported spinning sphere has been investigated by Fairbank and Little at Stanford.[18] Another approach, untilizing an electrostatically supported spinning sphere is being developed at the University of Illinois Coordinated Science Laboratory.[19] It is likely in either case that the experiment will have to be carried out in a satellite in order to minimize support bearing forces and attain the required precession.

Gravitational radiation is a particularly interesting problem

since its theoretical and experimental status are both unclear. That is, there is neither unambiguous theoretical prediction of gravitational radiation nor certainty that gravitational radiation from galactic and extra galactic sources passing the vicinity of the earth should be of sufficient strength to be detected. In Chapter 5, Weber reviews briefly the theoretical grounds for beliving gravitational radiation exists and describes a bold and skillful attempt by himself and his collaborators to observe such radiation.

3.3. Fundamental Precision Measurements

These are the precision experiments prerequisite to any theory of gravity. Three of these: The Eötvös experiment on the local equivalence of gravitational and acceleration fields, the mass isotropy experiment, and the charge unequality experiments have been discussed earlier in a paragraph on the experimental foundation of general relativity. There are additional precise experiments concerned with equally fundamental characteristics of space which are described below.

3.4. Gravitational Effects Predicted or Suggested by Alternative Theories and Hypotheses

In light of the mathematical complexity of general relativity, the difficulty of deriving observable predictions from this theory, and the small number of such predictions, another approach to experimental gravitation is the attempt to detect and measure effects not predicted by general relativity but which are

supported by reasonably plausable arguments or by alternative theories. Dicke has initiated several experiments along these lines, both by his work in indicating the scope of alternative theories equally well supported by experimental evidence and by the variety of experiments which he has suggested (Chapters 7, 8 and 12).

Several of these experiments have centered around the effort to observe effects of a preferred reference frame for gravitational field effects as might be expected in accordance with Mach's principle. In one sense, it is clear that there is a preferred frame of reference given by Newton's first law, that is, the inertial frame with respect to which bodies not subject to forces continue in uniform motion (or rest) and with respect to which a gyroscope maintains a constant orientation. Observationally, this reference frame with regard to rotation turns out to be coincident with the frame determined by the distant fixed stars up to an arbitrary uniform relative motion (excluding the small not yet observed relativistic effect mentioned previously in connection with gyroscope precession). Furthermore, we can single out an unambiguous fixed preferred reference frame by choosing one which is static relative to the average motion of the receeding galaxies in the expanding universe, that is, the frame for which distant matter exhibits no net rotation and the recession velocities of galaxies are isotropic, independent of the direction of observation.

The tantalizing question is then, are there any effects on gravitation, strong, or weak interactions due to uniform motion relative to this "cosmological" coordinate system? The search for such effects on strong interactions (electromagnetic propagation) is represented by the attempts to observe ether drift effects by Michelson and Morley[20] and their successors, and by the modern experiments of Townes, Cedarholm, Bland, and Havens[21] and of Turner and Hill.[22] The experiment of Turner carried out with resonant absorption of Iron 57 gamma rays using Mössbauer effect for line narrowing imply a motion of the laboratory relative to a "special frame" of less than 900 cm/sec.

An attempt to detect directly an effect of motion relative to the coordinate system of the fixed stars on the strength of the gravitational interaction has been made by Hoffmann[23] by monitoring the period of a precision pendulum gravimeter as the earth moves in its orbit about the sun. Since the elliptical motion of the earth is superimposed on the motion of the sun in the galaxy, and ultimately relative to the cosmological coordinate system, the square of the velocity of the earth relative to this system is modulated with a period of a year. This experiment, with an accuracy of 2 parts in 10^8, implies that if gravitational attraction is affected by this motion with a term quadratic in (v/c), then the coefficient of this term is less than 1.

A search for subtle gravitational effects by direct measure-

ments of g present a struggle for precision and stability. While a pendulum is capable of very precise measurement, it is subject to drift and aging. Another approach, less precise, but with potentially much geater long term stability, is that of dropping an interferometer plate for making "absolute" measurements of g. This approach has been developed by Faller.[24] Other possible observable effects due to a varying gravitational constant and to scalar gravitational waves are discussed in Chapters 8 and 12.

3.5 General Relativity in Astrophysics

The departures in the metric from that of flat space due to a mass distribution are of the order of

$$\frac{Gm}{rc^2} \sim \frac{Gr^2\rho}{c^2} \tag{4.1}$$

where ρ is the density.

For ordinary laboratory sized objects this is approximately 10^{-25}. At the "surface" of a proton it is 10^{-39} and at the surface of the sun 10^{-6}. It is likely that in advanced stages of stellar evolution, the core of a star becomes a degenerate neutron gas, with a very high density. This number becomes nearly 1. In such cases, the distortion of the metric may lead to large effects on the prediction of stellar structure theory. This is discussed in Chapter 10 in connection with the collapse of a "superdense" dwarf star.

Gravitation theory may also play an important role in the evolution of stars if the gravitational constant varies in time as

suggested by Dicke in Chapter 8.

3.6 Cosmological Effects of Gravitational Theory

What are the boundaries of the universe? Is it open and infinite or finite in extent? Is it curved into closure? Einstein's equations permit all three possibilities. Wheeler argues in Chapter 15 that Mach's principle imposed as a boundary condition to the equations of general relativity restricts solution to those of a closed (positive curvature) universe.

Observationally, the question is unsettled. The present state of the observational evidence for various theoretical models has been discussed in detail by Sandage.[25] He summarizes the observational data for four cosmological phenomena.

A Minimum Size for Distant Galaxies

This is illustrated for a two-dimensional closed space (a spherical surface) in Figure 2.

The relationship between the angular diameter and magnitude of distant galaxies for a Friedman model of the universe for several different values of the acceleration parameter q is given in Figure 3. For a steady-state universe the angular diameter approaches asymptotically to a constant value. For a closed universe ($q > \frac{1}{2}$) the angular diameter of distant galaxies exhibits a minimum size. This minimum size occurs at a distance greater than the distance that for which the 200 in. telescope can be of use. Maybe in the future one will be able to determine if a minimum exists (and hence be able to determine whether the universe is closed or not).

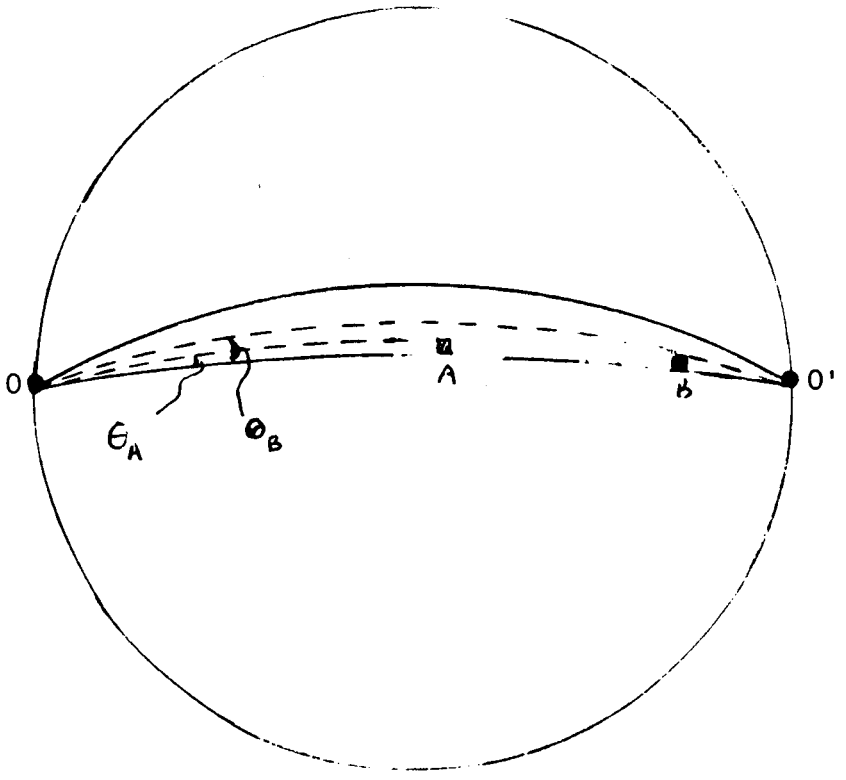


Figure 2. On a closed two dimensional spherical surface an object at position A subtends a minimum angle for an observer at 0. The same sized object at B subtends a larger angle as shown. The lines drawn on the suface are the geodesics (lines of shortest distance) for a sphere. 0' is the antipodal point.

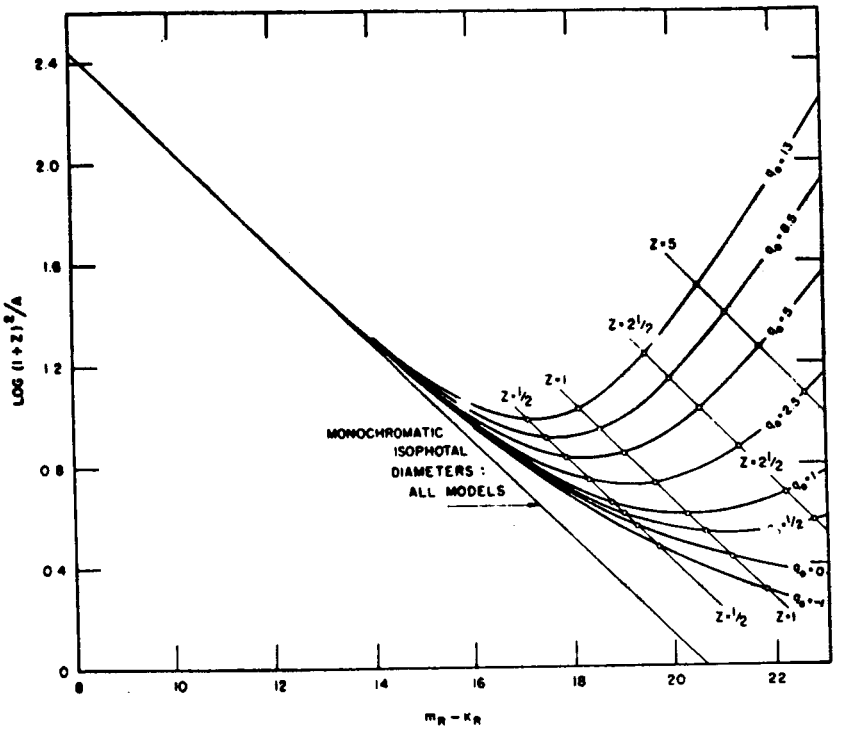


Figure 3. Metric diameters of either individual galaxies or of clusters of galaxies computed on the magnitude system of the brightest cluster galaxy. No normalization of the ordinate to agree with observational data has been made. The steady-state model (q_o = - 1) and seven exploding models are shown. Lines of constant red shift are given. The straight line gives the isophotal diameters for all models by $\theta_i = \frac{\text{const.}}{A}$ (A.R. Sandage, Astrophysical Journal 133, 379 Figure 7 (1961)).

Variation in Density of Galaxies with Distance

Assuming that galaxies are distributed uniformly in the universe on the average, in a curved universe the distribution would appear to be non-uniform. Unfortunately, the data now available for the galaxy-count magnitude relation are not adequate to lead to any significant conclusion concerning the various cosmological models.

Non-linearity in Redshift Observation

The observation that spectral lines from distant galaxies appear to be shifted in proportion to their distance from us has led to the model of the expanding universe with velocity of recession of galaxies proportional to their separation

$$v = \frac{R}{T} \tag{5.1}$$

where T is the reciprocal of the Hubble constant. T is currently determined to be 1.3×10^{10} years.[25]

The exact prediction of variation of velocity of recession with distance depends on the cosmological model taken. For example for a closed (Friedman type) universe ($q > \frac{1}{2}$) the observed recessional velocity increases more rapidly with distance.

One of the original purposes argued for building the 200 inch Palomar telescope was to answer this question. The ability of this telescope to distinguish distant objects from the sky background is limited up to the 23rd magnitude. For a normal galaxy this limitation corresponds to a distance of 4.5×10^9 light years. Unfortunately due to the increase in our determination of the scale of the universe this range has not been adequate to resolve this question. Figure 4 summarizes the observations with respect to

the universe for several values of the acceleration parameter.

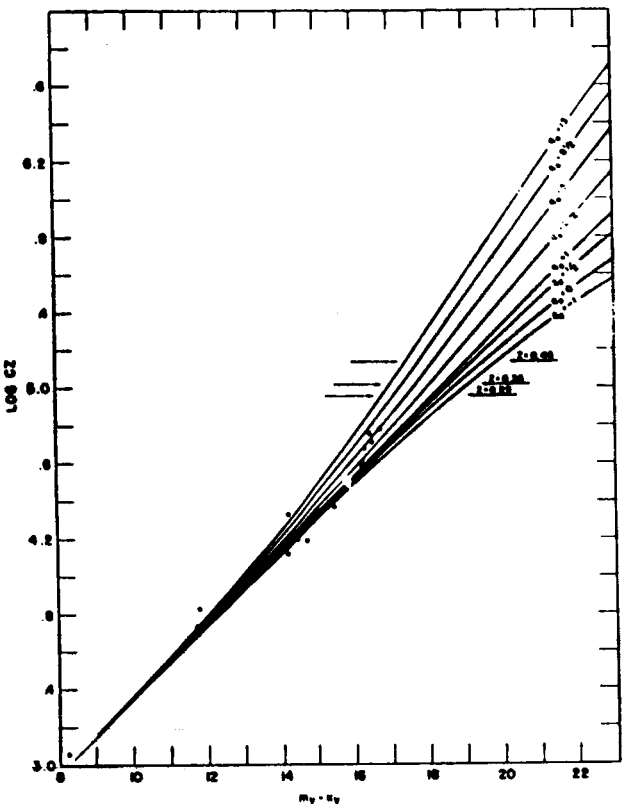


Figure 4. The theoretical red-shift-magnitude relation for various values of the acceleration parameter q_o. For a steady state universe $q_o = -1$, and for an open Euclidian universe $q_o = \frac{1}{2}$. For all models with $q_o > \frac{1}{2}$, the universe is closed. z is the red shift $\Delta\lambda/\lambda_o$ and c is a calibration constant on the absolute magnitude scale. Magnitudes are on the standard V system. Data for 18 clusters of galaxies are plotted as given by Humason, Mayall, and Sandage.[26] Arrows are placed at the observed red-shift values for these distant clusters whose magnitudes are not yet available. The magnitude gives a measure of the distance, for a Euclidian space the relation is given by $R \propto (\text{magnitude})^5$. (A.R. Sandage, Astrophysical Journal 133, 367 Figure 2 (1961)).

Time Scales

The comparison of time scales drawn from cosmological models, stellar evolution, and the dating of rocks and meteorites has proved to be one of the most successful methods for evaluating cosmological models. This is discussed extensively in Chapter 8 in connection with a time varying gravitational constant.

4. Quantization of General Relativity

Soon after the introduction of quantum theory, the possibility of quantizing general relativity arose. This possibility, to date, has not been realized. There are immense difficulties in quantizing general relativity. Many of these difficulties come from (1) the non-linear nature of the field equations and from (2) the covariance requirement.

1) The gravitational field equations are intrinsically non-linear and this non-linearity cannot be removed. The source of gravitational field is energy itself. Since gravitational fields also carry energy, the superposition of two gravitational fields will not be merely the sum of the two fields. Non-linear effects are important and this non-linearity is associated with the nature of the field itself and cannot be removed.

2) As we have seen, the Einstein field equations admit solutions up to an arbitrary number of coordinate transformations. That means, unless one arbitarily sets up a set of coordinate conditions, one cannot solve the Einstein field equations. Hence, without coordinate conditions Einstein's equations are empty, and quantization procedures cannot be carried out consistantly. To quantize

Einstein's field equations with coordinate conditions, on the other hand, is to deny the principle of covariance. This difficulty is particularly summarized in Chapter 14.

Now we shall ask if there is ever a need to quantize the gravitational field. The self gravitational energy, ϵ_{self}, of a proton (assuming a nearly uniform density) is

$$\epsilon_{self} = \frac{GMp^2}{R} \cong 10^{-43} \text{ ergs} \tag{6.1}$$

where R is the radius of a proton ($\cong 10^{-13}$ cm). Now, if we start to measure the rest energy of a proton at the beginning of the universe ($\sim 10^{17}$ seconds ago), the uncertainty $\Delta\epsilon$ in the measurement is given by

$$\Delta\epsilon \cong h/t = 10^{-44} \text{ ergs} \tag{6.2}$$

Hence $\Delta\epsilon \sim \epsilon_{self}$. This means that <u>in principle</u> a quantized gravitational field may not be a useful concept. The general relativitic effect seems to be non-microscopic even in principle.

Another reason that one wants to quantize the gravitational field is perhaps, one might think that the gravitational field may interact in the fundamental particles level and atomic level for which the quantum theory is for. According to a theorem in a Riemannian geometry, one can locally transform away all gravitational fields. Unless the curvature of space has such a large value that over the atomic dimension (10^{-8} cm) or the dimension of an elementary particle (10^{-13} cm) it changes substantially, one can just treat the interaction of microscopic matter with the

gravitational field semi-classically. Enormous concentrations of matter are needed in order to produce a field gradient so large that quantum effects come in. Such concentration of matter seems to be beyond our grasp of imagination. Even in the center of neutron stars whose masses are very close to the critical mass, such conditions are not realized.[27]

For the above reasons, the outlook for a useful, consistant quantized, non-linear, gravitational field is poor. However, several promising attempts to quantize linearized gravitational theory have been made. Among these are those done by Thirring[28] and Feynman.[29]

Thirring's approach to the problem of classical fields is based on an action principle from which both the field equations and the equations of motions are derived. In doing so, the existence of a conserved energy-momentum tensor is guaranteed. Such a tensor is needed because one can then require the energy of the system without interaction to be positive definite. He investigated three fields, the scalar, the vector, and the tensor fields; the source of all these fields is due to the mass.

The vector field is excluded from the very beginning because

in such a theory, like particles repel each other and this is contradictory to the behavior of the gravitational field.

The scalar field theory is very similar to the tensor field theory in many respects, it did not give the correct value for the advancement of the perihelion of mercury around the sun.

Thirring considered only the weak field limit of the tensor field theory. First, the space is assumed to be Euclidian. However, this Euclidian character is not observable because measuring clocks and rods will vary in size according to the field strength. What one actually observes is a Riemannian character of the space-time continuum. There also exists a limiting velocity, which need not be a constant. However, by a coordinate transformation this limiting velocity may be made to be the same everywhere, and then the space becomes Riemannian.

Second, the trajectories of particles are descibed by equations in which the mass of these particles disappear. If a coordinate transformation is carried out as above, the equations of motion become geodesic equations.

Third, his theory preserves gauge invariance character only when a coordinate transformation is carried out to transform the Euclidian geometry he started with into a Riemannian geometry.

Fourth, all results of Einstein's field theory to the order of v^2/c^2 are reproduced faithfully in the tensor field theory.

In addition Thirring's theory predicts the principle of equivalence. Also, the principle of general covariance is replaced by a gauge invariance principle which also serves as a guide for finding the non-linearities. Moreover, his theory predicts that gravitational forces are attractive (which is not predicted by Einstein's theory).

To within all experimental verifications of Einstein's theory of gravitation, we regard Thirring's theory (and a number of others) to be as satisfactory as Einstein's theory.

Feynman's work[29] extends Thirring's theory to the quantized version. He assumed the quantum of the gravitational field to be a massless spin 2 particle, and the source of such a field to be the energy. This last requirement makes the theory non-linear. Quantization can be carried out in the lowest order. Although in all calculations divergences occur, in certain cases renormalization can be carried out. The results of his theory are identical with those of Einstein's theory in the lowest order.

More recently, Wigner and Salecker[30] have investigated the meaning of measurements in general relativity. They found that the concepts utilized in general relativity are usually macroscopic in nature, whereas the applicability of quantum mechanics is limited to microscopic phenomena. Hence it appears that the quantum effects of the gravitational field may not have an operational meaning.

References

1. A. Einstein, Ann.d.Physik 49, 769 (1916); also an English translation in The Principle of Relativity, pp. 109-164, Dover Publications, 1923.

2. R.V. Eötvös, D. Pekar, and E. Fekete, Ann.d.Physik 68, 11 (1922).

3. R.H. Dicke, Sci.Am. 205 no.6, 84 (1961).

4. A. Papapetrou and E. Corinaldisi, Proc.Roy.Soc. (London) A 209, 259 (1951).

5. V.A. Fock, The Theory of Space-time and Gravitation, translated by N. Kemmer, Pergamon: New York, 1959.

6. H. Weyl, Raun-Veit-Naterie, 1st ed., Springer: Berlin, 1918.

7. V. Beltram-Lopez, H.G. Robinson, and V.W. Hughes, Bull.Amer.Phys. Soc. 6, 424 (1961).

8. J.C. Zorn, G.E. Chamberlain, and V.W. Hughes, Bull.Amer.Phys.Soc. 4, 63 (1961).

9. A.M. Hillas and T.E. Cranshaw, Nature 184, 974 (1959).

10. L.I. Schiff, Amer.J. of Phys. 28, 340 (1960).

11. C. Brans and R.H. Dicke, Phys.Rev. 124, 925 (1961).

12. U.J. LeVerrier, Annales de l'Observatoire de Paris 5, 1 (1859).

13. S.Newcomb, Astronomical Papers of the American Ephemeris 1, 472 (1882).

14. G.M. Clemence, Reviews of Mod.Phys. 19 no.4, 361 (1947).

15. W.S. Adams, Proc.Nat.Acad.Sci. 11, 382 (1925).

16. R.V. Pound and G.A. Rebka, jr., Phys.Rev.Letters 4, 337 (1960).

17. J. Brault, Bull.Am.Phys.Soc. 8, 28 (1963).

18. L.I. Schiff, Phys.Rev.Let. 4, 215 (1960) and Proc.Nat.Acad. Sci. 46, 871 (1960).